

## The study of phenomenon of ozone layer depletion as a physical process<sup>†</sup>

M A K Yousuf Zai\* and Jawaid Quamar

Institute of Space and Planetary Astrophysics (ISPA), University of Karachi, Karachi 75270, Pakistan

E-mail : ayubzai@yahoo.com

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**Abstract** : Ozone ( $O_3$ ) is the most important trace constituent of the stratosphere. Incidents such as the industrial emissions of CFCs, supersonic transports, and volcanic eruptions provide a most visible example of global unbalance in our natural ecology. Among other scientific and socio-economic fallouts from this, the aggravation of the phenomenon of ozone layer depletion (OLD) is particularly disturbing. Events like 1987 Montreal Protocol explain the magnitude of the threatening status of the OLD. For OLD analysis, to overcome doubts of the effectiveness of the mathematical formulation against the empirical relation, this work studies the formulation of the phenomenon of OLD as a physical process, with special reference to the stratospheric region of Pakistan. This paper establishes the results of preliminary calculations reported elsewhere, confirming quantitatively the phenomenon of OLD.

**Keywords** :  $O_3$  layer depletion, stratosphere of Pakistan, assessing structure of  $O_3$  variations

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### Introduction

$O_3$  is one of the species produced by the photochemical reactions in the stratosphere. Ozone  $O_3$  absorbs almost all solar UV radiation of wavelengths less than  $\sim 320$  nm (nanometer) and prevents harmful radiation from arriving at the earth's surface. Without the  $O_3$  shield, even lower animals could not survive, since the harmful UV radiation would destroy chromosomes in the cell nucleus, thus prohibiting cellular multiplication. Exposure to the proper amount of UV radiation has benefits such as production of Vitamin D in the human body but excess exposure results in harmful effects such as sunburn, skin cancer, cataract, etc. Nature protects life on the surface of the earth by maintaining proper amount of  $O_3$  in the middle atmosphere. Thus, if the amount of stratospheric  $O_3$  should change, life on the earth, as we know it, would change. There are several human activities that may cause significant changes in the stratospheric  $O_3$ .

These changes include  $NO_x$  emissions from Super Sonic Transport (SST) aircrafts, the release of chlorofluorocarbons (CFCs) from aerosols spray and cans, and refrigerators, and

increase of  $N_2O$  in the atmosphere due to fertilizing agricultural fields [1–3].

The biospheric resource consumption in industrially developed countries seems to exceed that in the developing world. This is resulting, among other adverse effects, in the production of hundreds of thousands of anthropogenic substances, 'unnatural' chemicals dubbed xenobiotics which are foreign to living organisms. Many of these have found their way into the biosphere and have been classified as toxic. Such environmental toxicity poses potential hazards to the entire living environment, being capable of poisoning even various 'foodchains' of oceans and the surface of earth [4]. Apart from sulphate compounds and halons etc., CFCs have in particular, wrought out perturbations of the biosphere among which the  $O_3$  layer depletion (OLD) stands first [5]. Due to the destruction of  $O_3$  blanket and the resulting artificial climatic change —global warming [6]—we are facing abnormally high incidences of UV-B radiation on the surface of earth [7–10]. Numerous facts have been found which testify to the destructive impact of UV-B radiation due to OLD on aquatic organisms [11–17].

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\*Corresponding Author

The legal document signed by as many as 24 countries on O<sub>3</sub> depleters, subject to ratification in 1987 (and known as the Montreal Protocol), consists of main points such as freezing CFC consumption to 80% of the levels of 1986 and reducing CFC uses to 50% of the 1986 levels [15,20]. Likewise, a special session of the UN General Assembly took place in June 1997 to discuss the results achieved during the 5-year period after the Agenda 21 and Framework Convention for Climate Change. Here, it is worth noticing that the low level of carbon emissions in the former USSR countries is a result of declining economies. Again, in China and India, the emissions are low. But in view of the necessity of further industrial development in such countries, it is obvious that the total emissions are expected to increase in the near future [8,10] if we all maintain status quo. In other words, if the CFCs followed free-market growth until 2002, the Antarctic O<sub>3</sub> hole would be a permanent fixture through the 21-st century, instead of disappearing by 2050 as predicted in the Copenhagen 1992 scenario [10,18–21].

However, a firm connection between CFCs and the Antarctic O<sub>3</sub> hole (O<sub>3</sub> layer depletion or OLD) has been established. In this context, a basic and decisive role belongs to understanding the phenomenon of OLD as a physical process. In particular, a natural and more interesting question is about the effects of OLD in regions other than that of Antarctic itself, for instance, impacts with reference to Pakistan's atmosphere [22–26].

The present study supplements our earlier paper [27], based on data developed *via* observational programmes on the global O<sub>3</sub> detection network conducted under the auspices of UNO, and introduces here the problem of studying the contemporary variability of O<sub>3</sub> contents as a process.

Section 2 introduces some techniques to assess the behaviour of OLD prior to embarking on a fuller mathematical formulation. Section 3 then deals with the problem of estimating the principally important distribution parameters, while Section 4 attempts a formulation of OLD as a process. Finally, Section 5 concludes the communication.

## 2. Assessing the behaviour of stratospheric O<sub>3</sub> variations

As indicated in Section 1, to unite the empirical spirit with the mathematical formulation, we will base our considerations on the O<sub>3</sub> time events

$$\{X_i\}, \quad (i \in \{1, 2, \dots, 480\}). \quad (1)$$

This communication comprises of the monthly observations for Pakistan covering a period from January 1960 to December 1999. Existence of an acute problem is immediately confirmed by a look at the variation of ozone depths against time

(Figure 1) which clearly shows that the depletion far exceeds the restoration of O<sub>3</sub> in the earth's stratosphere. We first try

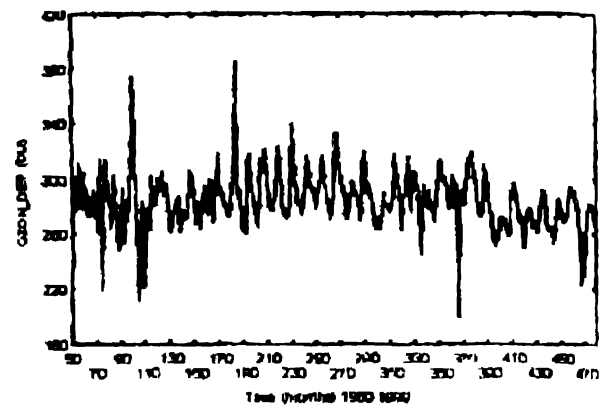


Figure 1. Variations of ozone depths *versus* time, showing that the depletion far exceeds the restoration of O<sub>3</sub> in the earth's stratosphere

to look into the obvious general character of this 'process' and assume that the 480 observations possess statistical independence, though (contrary to conventional thinking) they may not share the same probability distribution. Because of its more powerful character {e.g. over that of the familiar Chi-square test ( $\chi^2$ -test)} and its suitability even for small sample sizes, we next invoke the technique of Kolmogorov-Smirnov (KS) goodness-of-fit test for comparing an observed sample space with the theoretical distribution [22].

Now our null hypothesis ( $H_0$ ) in the KS-test is that the observed distribution does not differ significantly from the theoretical distribution. This test may be taken to mean that it pinpoints the maximum absolute difference,

$$W = \max |F_0 - F_e|, \quad (2)$$

between  $F_e$ 's and  $F_0$ 's. Thus, we must calculate a cumulative expected frequency  $F_e$  expressed as a proportion of the total for each observed frequency  $F_0$  in the series (1). In Table 1, we find that the O<sub>3</sub> depth of about 290 DU occurs at the maximum difference of about 0.042 [eq. (2)], recalling that the original data are measured in Dobson unit [1 DU =  $10^{-3}$  cm of O<sub>3</sub> at STP of the atmosphere] [27]. If we can argue that this absolute difference is significantly large, then we would reject the null hypothesis ( $H_0$ ) and the contention that the depth of O<sub>3</sub> layer is from a completely normal distribution.

We can perform the KS-test to get our  $W$  as follows. As usual, we choose for the purpose, a confidence level of 95% so that the significance level is determined by

$$95\% = (1 - \alpha)100\% \Rightarrow \alpha = 0.05. \quad (3)$$

Thus, as the size of sample space of O<sub>3</sub> concentration is  $n \geq 35$ , the  $W$ -value as calculated from KS-tables comes to be  $1.36/\sqrt{480} = 0.062$ . As this value exceeds the  $W$  given by

eq. (2), we accept  $H_0$  and assume that OLD could be simulated by sampling from a normal distribution with a

Table 1. Kolmogorov-Smirnov goodness-of-fit test.

Ozone oth	Cumu % observed	mmu % expected
190	0 000	0001
200	0 0283	8509
210	0 0283	0090
220	0 6250	.0603
230	1.2500	8180
240	1 8750	.3210
250	2 7083	.3508
260	7 500	4544
270	21 0417	3817
280	44.7917	6443
290**	66 8750	6728
300	83.7500	7249
310	92.2917	9954
320	97 5000	7779
330	83 3333	.0807
340	99 3750	7924
350	99 5833	9631
360	99 7917	9994
370	100 000	9999
infinity	100 000	0 000

\*\* Indicates the depth of  $O_3 = 290$  DU which occurs at the maximum difference 0.042

mean  $\bar{X}$  and standard deviation  $\sigma$ . Acceptance of the hypothesis  $H_0$  i.e. the goodness-of-fit to a normal distribution is further corroborated by the KS-plot shown in Figure 2.

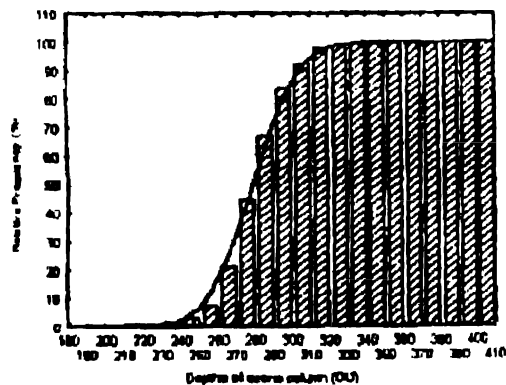


Figure 2. Kolmogorov-Smirnov (KS) goodness-of-fit for OLD to assess how well the data set appears to come from a normal distribution

As a further check of the correctness of the above finding, we may consider the scatter of the distribution relative to the size of the estimated mean revealed by the concept of coefficient of variation,

$$CV = \frac{\sigma}{\bar{X}} = 0.07. \quad (4)$$

Eq. (4) shows that the distribution possesses positivity and right-skewedness. Moreover, the sufficiently low value of the calculated  $CV$  indicates a good degree of normality obeyed by the different  $O_3$  depth events. The above numerical estimate indicates that just about 7.0% of the data is non-normal.

### 3. Estimating the size of stratospheric resident $O_3$ fluctuations

Among the standard classical and modern tools for parameter estimation, we may choose the MME (method of moment estimator) technique for our current context because of its several useful properties discussed in the standard literature [28-32]

To produce acceptable estimates, the followings are the most widely used methods :

- (i) The method of moments,
- (ii) The method of Maximum likelihood estimation.

In this communication, we use the method of moments to estimate the population parameters, such as mean of the population from the sample distribution, standard deviation, and variance of the population. The moments are essentially functions of the sampling distribution of a random variables. The method of moments is an estimation technique which suggests that the unknown parameters mentioned above should be estimated by matching population (or theoretical) moments (which are functions of the unknown parameters) with the appropriate sample moments. Method of moments estimators (mme) are based on the sample idea of equating the sample moments based on the data with the moments of the probability distribution.

#### The $r$ -th moment of a distribution

The  $r$ -th moment of a random variable  $X$  is often denoted by

$$\mu'_r = E(x^r). \quad (5)$$

The first moment of  $x$  is called the *mean* of  $x$  and is usually denoted simply by  $\mu$ . That is .

$$\mu = \mu'_1 = E(x). \quad (6)$$

The formula for the  $r$ -th sample *moment* is

$$\hat{\mu}'_r = \frac{1}{n} \sum x_i^r. \quad (7)$$

Thus, the method of moments estimators of the mean of the distribution is

$$= \frac{1}{n} \sum_{i=1} x_i = \bar{x}. \quad (8)$$

The quantity

$$\frac{1}{n} \sum_{i=1}^n x_i =$$

is called the sample mean.

The  $r$ -th central moment of a distribution :

$$\mu_r = E[(x - \mu)^r]. \quad (9)$$

The second central moment of the random variable  $x$  is called the *variance* and is usually denoted by  $\sigma^2$ . That is

$$\sigma^2 = \mu_2 = E[(x - \mu)^2]. \quad (10)$$

The square root of the second central moment is called the *standard deviation* of  $x$  and is usually denoted by  $\sigma$ .

The mathematical expression for the  $r$ -th sample Central moment is

$$\mu_r = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^r. \quad (11)$$

Thus method of moment estimator of the variance of this distribution is

$$\hat{\sigma}_{\text{mme}}^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 = S_m^2 \quad (12)$$

and the moment estimator of the standard deviation of this distribution is  $\hat{\sigma}_{\text{mme}}$ .

If the average value of an estimator equals the population parameter, the estimator is said to be *unbiased*. If the average value is less or greater than the population parameter, the estimator is said to be negatively or positively biased or simply biased.

A *normal distribution* is characterised by its mean ( $\mu$ ) and variance ( $\sigma^2$ ) or standard deviation ( $\sigma$ ).

$$\mu = \sum x_i / n (= \bar{X}) = 283.65 \text{ DU}, \quad (13)$$

which can be taken to provide an estimate of the population mean  $\mu$

$$\tilde{\sigma}^2 = \frac{\sum (x_i - \bar{X})^2}{n} = 386.52. \quad (14)$$

Plugging values in eq. (14), we see thus that the standard deviation may be taken as

$$\tilde{\sigma} = 19.66 \text{ DU}. \quad (15)$$

The above (point) estimate for  $\mu$  is of little value unless we know how accurate the estimate is likely to be. According to the central limit theorem, we know that for sufficiently large  $n$  (here  $n$  being 480), the sampling distribution of the sample mean  $\bar{X}$  is approximately normal with

$$E(\bar{X}) = \mu, V(\bar{X}) = \sigma^2 / n. \quad (16)$$

$\bar{X}$  may be considered as the best estimator of  $\mu$  in view of the relation (18) and because it is easy to show that  $\bar{X}$  has

the smallest variance among all unbiased estimators of  $\mu$ . Now, as  $\bar{X}$  is approximately normal, one way to ascertain the accuracy of our  $\hat{\mu}$  consists of constructing a large-sample  $(1 - \alpha)100\%$  classical confidence interval (CI) for the population mean. Inserting

$$\hat{\mu} = \bar{X}, \sigma_{\bar{X}} = \sigma / \sqrt{n} \quad (17)$$

into the usual expression for the confidence interval gives

$$\bar{X} \pm z_{\alpha/2} \sigma_{\bar{X}} \approx \bar{X} \pm z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right) \quad (18)$$

where  $z_{\alpha/2}$  is the value of the standardised normal variable  $z$  that locates an area of  $\alpha/2$  to its right. As  $n \geq 50$  in our case, the approximation (18) for CI is quite satisfactory. In fact, when the value of population standard deviation, say  $s$  is unknown, the sample standard deviation  $\sigma$  may be used to approximate  $s$  in eq. (20) for the CI.

Now, using normal distribution tables, we find that the area to the left of  $z$ -value  $a$ , viz.  $\phi(a) \equiv (1 - \alpha/2) = 0.975$  yields  $a = 1.96$ , so that  $z$  lies between  $-1.96$  and  $1.96$ . In other words, 95% of OLD sample of size  $n$  being considered, lies between confidence limits described by the following range :

$$\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}}. \quad (19)$$

In view of eq. (17), the population mean  $\mu$  thus lies in the interval

$$\bar{X} \pm 1.96 \times 19.66 / \sqrt{480} \equiv \bar{X} \pm 1.76. \quad (20)$$

The confidence limits (19) are generic in that they differ from sample to sample. For the specific O<sub>3</sub> depth sample (1) for which  $\bar{X} = 283.65$ , the confidence limits are given by

$$81.89 < \mu < 285.41, \quad (21)$$

with the probability of such an occurrence of the population mean being

$$P(\bar{X} - 1.76 < \mu < \bar{X} + 1.76) = 0.95. \quad (22)$$

In other words, we know that our particular sample has 95% probability that the population mean  $\mu$  will lie between the indicated confidence limits.

Eq. (13) lends credence to a physical picture of the time events (1) as a stationary process in that Figure 1 shows a dominance of time events resembling our  $\hat{\mu}$ . This in turn, necessitates a search for the possibility of further structural pattern, which we take up in the next section.

#### 4. Conformation of stratospheric resident O<sub>3</sub> fluctuations

The fluctuations in depth of O<sub>3</sub> layer, arising from its interactions with various atmospheric processes, measured at different points of time during the years 1960–1999 can be thought of, at least in part, as emerging from randomness

as a particular realisation of a stochastic process [28–30, 34, 35]. We have decided to ignore 'fractality', 'multifractality', 'deterministic chaos' [33] for mathematical tractability. The question is how to sift the complexity of the restoration and depletion of the stratospheric resident  $O_3$  in order to understand the inherent stochastic character and to discover interesting structural properties of OLD. The trend set by newer developments such as simulation, symbol dynamics, chaos research, *etc.* in atmospheric sciences, meteorology — as contrasted with the age-old deduction-induction dichotomy — suggests that we invoke the idea of a controlled model, the notion of simpler repeatable representation, to help us define and crystallise the realistic situation of the complex 'process' in hand. For an illustration, such an attempt at deducing infer about the physical mechanism generating the series of  $O_3$  events from the processes, may be partially likened to a population from a sample in the setting of classical statistical analysis.

As already indicated, for the sake of convenience, we may treat the process (1) as a linear phenomenon rather than a nonlinear one. Thus, in keeping with the spirit behind the most general as well as the prime example of *ab initio* mathematical formulation of a process *viz.* generalised linear modelling, an immediate candidate seems to be multiple regression approach but one in which some or all the explanatory variables are 'lagged' values of a 'time-dependent' random variable  $X_t$ . Thus, taking into account the mutually regressive relationship between random variables  $X_t$  — defining the space (1) — arising from the temporal interdependence of  $X_t$  on its predecessor  $X_{t-1}$ , we naturally land on a formulation of  $X_t$  as a linear combination of its two immediately preceding values, which in turn, readily yields the following special case of a generic multiple regression model :

$$X_t = \alpha_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \dots + \beta_s X_{t-k} + \gamma_t, \quad (23)$$

where  $k$  may obviously be dubbed as the order of the model (23), the value  $x_{t-k}$  (of the random variable  $X_{t-k}$ ) can be designated the lagged value of  $x$  at time  $(t-k)$ , and  $\gamma_t$  may be taken to stand for the possible white noise over the main signal represented by other terms in eq. (23). For mathematical tractability (so as to avoid further constraints on the system, such as the imposition of the assumption of moving-average hypothesis), we ignore here other terms in eq. (23) and settle down to a model equation

$$X_t = \beta_1 X_{t-1} + \alpha_t \quad (24)$$

for the description of the time-dependent history of  $O_3$  depths. The crucial assumption for our model (24), other than those already indicated, is that  $\alpha_t$ 's at different  $t$  are independent *i.e.*  $\alpha_t$  is independent of  $\alpha_{t-1}$ ,  $\alpha_{t-2}$ , .... This, in

turn, implies that  $\alpha_t$  is independent of  $X_{t-2}$ ,  $X_{t-3}$ , ... too (this being an expression of the fact that eq. (24) is self-regressive of order 1).

Notice that the model (24) is kind of conditional regression in that at time  $t-1$ , when  $X_{t-1}$  is fixed, eq. (24) is a regression model. In addition to the completely dependent constituent (given by  $\beta_1 X_{t-1}$ ) of  $X_t$  in the model (24), we have another constituent of  $X_t$  which happens to be independent of  $X_{t-1}$  (given by  $\alpha_t$ ). At time  $t-1$ , as  $X_t$  is an unknown random variable, so is  $\alpha_t$ , obeying a certain distribution. As soon as  $X_t$  is observed and known at time  $t$ ,  $\alpha_t$  no longer remains a random variable but gets fixed, which can then be computed by

$$\alpha_t = X_t - \beta_1 X_{t-1}. \quad (25)$$

Eq. (25), apparently just a different form of our given model (24), resulting from a consideration of the 'orthogonal decomposition' just discussed, provides us with an interesting interpretation. As  $\{X_t\}$  is a dependent series and  $\alpha_t$  is an independent one, we may consider the model (24) as a device to reduce a dependent data set into an independent one (accomplished by removing from  $X_t$  the part that depends on  $X_{t-1}$ ). An immediate implication is that the assumption of independence stipulated at the start of Section 2 is favoured by our proposed model (24).

Next, we must bother ourselves with the job of estimating parameters figuring in our model (24). For concreteness, we may assume that  $\alpha_t$  possesses a normal distribution :

$$\alpha_t \sim \text{NID}(0, \sigma_\alpha^2), \quad (26)$$

where NID stands for the phrase 'normally independent distributed'. As the model (26) is just a conditional regression, we can invoke the technique of conditional least squares to get estimates of  $\beta_1$  and  $\sigma_\alpha^2$  [36] :

$$\hat{\beta}_1 = \frac{\sum_{t=2}^n X_t X_{t-1}}{\sum_{t=2}^n X_{t-1}^2} = 0.549, \quad (27a)$$

$$\hat{\sigma}_\alpha^2 = \frac{1}{n-1} \sum_{t=2}^n \alpha_t^2 = \frac{\text{residual sum of squares}}{\text{number of residuals}} = 127.971 \quad (27b)$$

For the expressions above,  $\bar{X}$  has been subtracted from the data.

Notice that the estimate of the parameter  $\beta$ , figuring in our model (30) and given by eq. (27a), shows that

$$\hat{\beta}_1 < 1 \neq 0. \quad (28)$$

Thus the extent of the dependence of  $X_t$  on  $X_{t-1}$  (as measured by this parameter) is weak ( $\beta$  is small, for  $|\hat{\beta}_1| < 1$ ). Nevertheless, eq. (27a) does evince a relationship between  $X_t$  and  $X_{t-1}$  ( $|\hat{\beta}_1| \neq 0$ ). In other words, the process depicted by our  $O_3$  time events (1) is not just statistical but also not uncorrelated *i.e.* is stochastic, as we assumed here. Again, eq. (28) (small but nonzero  $\beta$ ) also shows that the process (1) is stationary — of course, corresponding to an stipulation of some suitable restrictions related to the white noise  $\gamma_t$  and the general terms in the original eq. (23) [37]. This stationarity has the important characteristic that mean, variance and correlation (covariance, in the language of usual statistical analysis) of the individual  $O_3$  events in the process (1) remain the same for all  $t$ . This information is encoded in the autocorrelation function

$$\rho_k := \lim_{n \rightarrow \infty} \hat{\rho}_k, \quad (29a)$$

$$\text{where } \hat{\rho}_k := \frac{\sum_{t=k+1}^n X_t X_{t-k}}{\sum_{t=k+1}^n X_{t-k}^2} \quad (29b)$$

is the estimate of the autocorrelation at  $k$  lags,  $\hat{\rho}_1$  being the autocorrelation (giving an estimate of the relation or dependence between values of  $X_t$ ) one lag apart or at lag one.

In an explicit corroboration of our argument at the beginning of this section, the line spectrum (periodogram) constructed in Figure 3 identifies the randomness in the  $O_3$

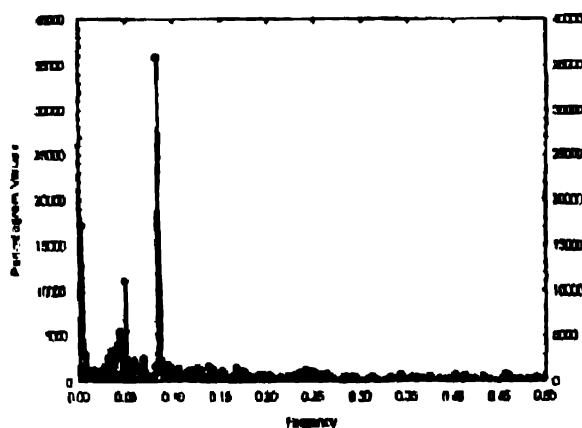


Figure 3. Periodogram displaying the stochastic process in the phenomenon of OLD.

depth process (1), it is in fact, speaks for a seasonality (or trend) among the given time events. Moreover, our line spectrum exhibits a predominance of positive over negative autocorrelations, *i.e.* a dominance of low-frequency amplitudes over high frequency ones. In other words, there is taking place a damping of  $O_3$  concentration over time.

Thus, the spectrum reinforces the assertion made at the beginning of Section 2.

Also, the scatter plot of  $X_{t-1}$  against  $X_t$  (see Figure 4) reveals, as expected, a rather complex dependence of the

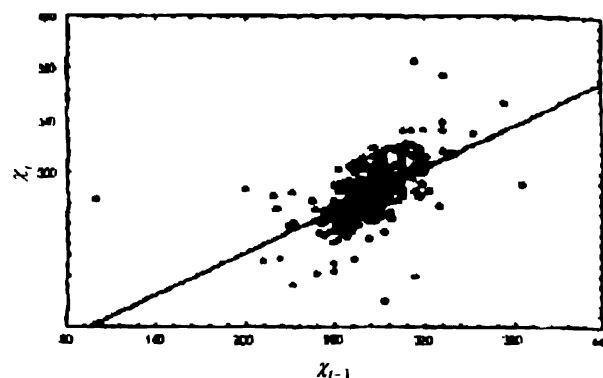


Figure 4. Scatter plot of  $X_{t-1}$  versus  $X_t$  elucidating rather complicated dependence of future and past values of ozone depths.

$$\hat{Y} = 127.971 + 0.549x + \varepsilon_t \text{ (Trend equation)}$$

future on the past, again as conjectured at the outset of this section. Clearly, there may exist serious temporal and spatial limitations of our model (24) as a representation of the real stratosphere. However, given this, the eddy diffusion in the model may be thought of as a radial diffusion of the  $O_3$  towards stratospheric region in question (such as Pakistan). In other words, this model (24) may be interpreted as a special case of the random walk [31] model of transportation of  $O_3$  flux. This nicely ties up with the critical geographical position of Pakistan — roughly covering the South Asia between  $\varphi \in [23.45^\circ\text{N}, 36.75^\circ\text{N}]$  and  $\lambda \in [61^\circ\text{E}, 75.5^\circ\text{E}]$  — and the large positive correlation between the potential vorticity deviations and  $O_3$  mixing ratios in the stratosphere [2].  $O_3$  depth fluctuations seem to be transported along with seasonal variations (*cf.* Figure 3) to Pakistan's atmospheric regions [26]. Moreover, the  $O_3$  layer variability forms an  $O_3$  filter in the passage of UV-B. This  $O_3$  filter appears to be transported to Pakistan *via* a vertical lifting followed by a horizontal mixing of  $O_3$  contents.

Our model (24) is further justified by numerous evidences. The residual analysis for eq. (25) graphed in Figure 5 amply demonstrates that the constructed model is reasonably adequate. Moreover, the correlational structure of the  $O_3$  process (1) (its correlation with itself) — determined by the autocorrelation function (29a) between the  $i$ -th observation and the  $(i + m)$ -th at various lags of 1, 2 or more periods — shows a sufficiently high degree of correlation (see Figure 6), the high orders of our model (24) indicating in turn, a good fit of eq. (24) to the temporal process (1). Furthermore, the error structure revealed by the autocorrelation for residuals of the  $O_3$  depth events exhibits a rather neat serial correlation (*vide* Figure 7). A comparison

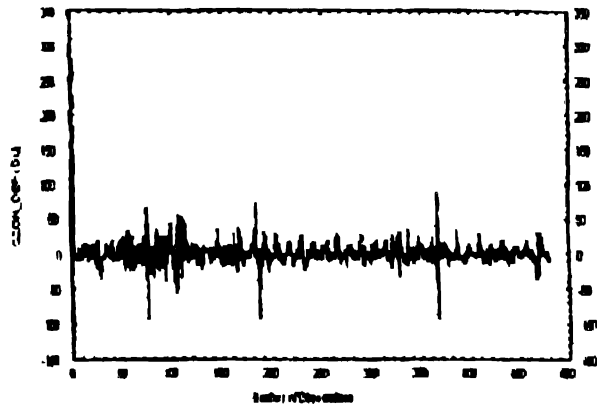


Figure 5. Residuals showing the adequacy of the constructed model of OLD for atmospheric region of Pakistan.

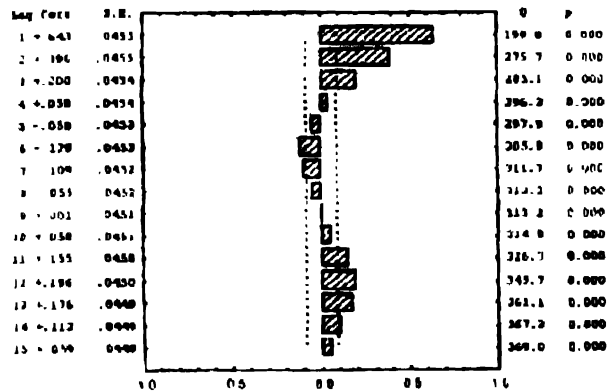


Figure 6. Autocorrelation function for OLD between  $i$ -th observation and the  $(i + m)$ -th giving high correlation

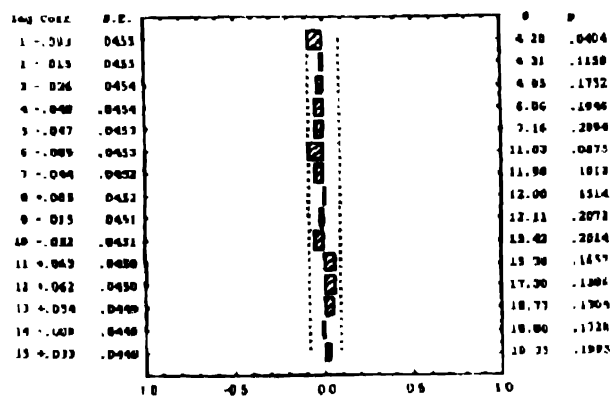


Figure 7. Autocorrelation function for the residuals of OLD, showing plainly the presence of high serial correlation between observed and predicted model values.

of observed  $O_3$  process described in Table 1 with the predicted one on the basis of eq. (24) comfortably establishes the validity of the constructed model (*cf.* Figure 8).

Plugging the estimates found above in the model (24) gives

$$\hat{x}_t = 127.971 + 0.549 x_{t-1}. \quad (30)$$

So the forecast for the  $O_3$  depth for the month of January 2000 (*i.e.* for the 481-st month reckoned from the year

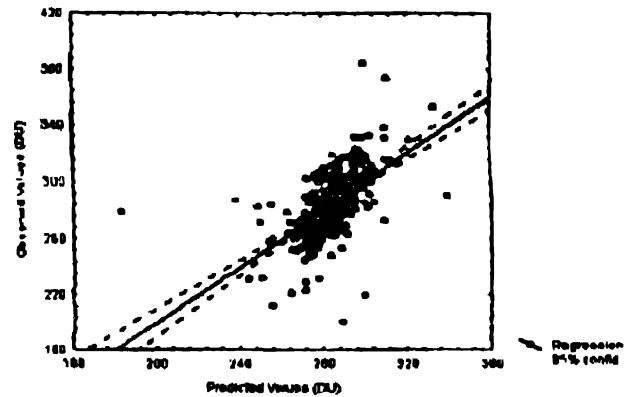


Figure 8. Comparison of observed and predicted values of OLD, establishing well the nature of constructed model (*cf.* text)

January 1960) is provided by the following equation on inserting  $x_{480} = 260$  DU in eq. (30) :

$$\begin{aligned} \hat{x}_{481} &= 127.971 + 0.549 x_{480} \\ &= 127.971 + 0.549 \times 260 = 270.711. \end{aligned} \quad (31)$$

It can be examined that the forecast accuracy is 4.12%, which is suitable for Pakistan's stratosphere.

## 5. Conclusion

As Section 1 shows, the phenomenon of OLD is a potential source of UV radiation on the surface of the earth. To meet this immediate threat, we require to properly gauge and monitor the impact of OLD on the present day environment. As argued in that section, for a systematic handling of the problem, we need to try to understand the nature of variations in the  $O_3$  concentrations in the stratospheric region of any specific area. Thus, as undertaken in Section 2, our calculations show that the process (1) possesses a good degree of normality, which is reasonable from the viewpoint of further analysis. However, it raises the question of the performance of Dobson spectrophotometers being used for recording the events at the detection centres, on the one hand, and of the actual configuration of the  $O_3$  depth probability distribution, on the other.

Next, utilising the size estimates of  $O_3$  concentration worked out in Section 3, Section 4 constructs and validates a linear self-regressive model, giving a forecast of  $O_3$  depths for Pakistan's stratospheric region with good forecast accuracy. In addition to explaining various physical features of the  $O_3$  phenomenon as a process, thus strengthening our earlier findings, it is quite interesting that such a forecast computation for  $O_3$  depths could lend insight into the very physical mechanism generating future events. To the best of our knowledge, the study presented in this paper does not seem

to have been undertaken in the published literature, in particular in a local/regional perspective. In fact, however, the question of O<sub>3</sub> depths is still unanswered in many respects e.g. how about a justification in treating the conditional regression as a regression, reduction in assumptions made for our model, a fresher scheme presumably superior to the suggested model, detailed analysis of the periodicities of OLD, modifying the model due to the role of aerosols in the OLD, etc.

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